Three-party qutrit-state sharing

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Abstract A three-party scheme for securely sharing an arbitrary unknown single-qutrit state is presented. Using a general Greenberger-Horne-Zeilinger (GHZ) state as the quantum channel among the three parties, the quantum information (i.e., the qutrit state) from the sender can be split in such a way that the information can be recovered if and only if both receivers collaborate. Moreover, the generation of the scheme to multi-party case is also sketched.

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I. Introduction

Secret sharing was proposed firstly by Blakley et al[1] in 1979. It can be depicted, in the simple case, as that a secret is divided by a sender into two pieces for two receivers. The secret can be reconstructed only if both receivers act in concert and neither of them can get anything about the original message solely. In 1999, this concept was generalized to quantum scenario by Hillery, Bŭzek, and Berthiaume (HBB)[2]. They endowed it a novel concept of quantum secret sharing (QSS). QSS is likely to play a key role in protecting secret quantum information, e.g., in secure operations of distributed quantum computation, sharing difficult-to-construct ancilla states and joint sharing of quantum money, and so on. Hence, after HBB's pioneering work, QSS, as an important branch of quantum communication, has so far attracted a great deal of attentions[3-28].

All the QSS works concentrate essentially on two kinds of problems. One deals with the QSS of classical messages (i.e., bits) [2-14]; another deals with the QSS of quantum information [2, 15-28], where the secret is an arbitrary unknown quantum state. The former is usually referred as quantum secret sharing (QSS); the latter as quantum state sharing (QSTS), which was first clearly termed by Lance et al. in 2004[19]. As far as QSTS is concerned, the first scheme was presented in 1999 by using a three-qubit or a four-qubit Greenberger-Horne-Zeilinger (GHZ) state for securely sharing an arbitrary unknown single-qubit state[2]. Soon later, Cleve et al.[15] investigated a more general quantum (k, n) threshold QSTS scheme. Bandyopadhyay[16] proposed a QSTS scheme using optimal methods in 2000, and Hsu[17] proposed other QSTS scheme based on Grover's algorithm in 2003. Recently, Li et al.[18] proposed an QSTS scheme for sharing an unknown single-qubit state with a multipartite joint measurement. Some QSTS schemes were implemented in cavity QED[21-22]. Zhang et al.[23] proposed a multiparty QSTS of an arbitrary unknown single-qubit state via photon pairs. Lance et al.[24] proposed other continuous-variable QSTS scheme via quantum disentanglement. Deng et al.[25-26] proposed two QSTS schemes for sharing an arbitrary two-qubit state based on entanglement swapping. Li et al.[27] proposed an efficient symmetric multiparty QSTS scheme of an arbitrary m-qubit state with m GHZ states. Very recently, Gordon and Rigolin[28] proposed two new QSTS protocols where the quantum channels are not maximally entangled states. Note that all these QSTS protocols except for that in Refs.[18,24], however, only treat single-particle *qubit* or multi-particle *qubit* state. In this paper, we will propose a QSTS protocol for sharing an arbitrary unknown single-particle *qutrit* state.

This paper is organized as follows. In section II, a three-party QSTS scheme is presented by using quantum entanglement swapping, and the scheme security is analyzed. In section III, the three-party QSTS scheme is generalized to a multiparty case. Finally, some summaries are given in section IV.

II. Three-party qutrit-state sharing scheme

Suppose there are three legitimate users. Alice is the sender of quantum information (i.e., an unknown qutrit state), Bob and Charlie are two agents. Any agent can reconstruct Alice's quantum information by collaborating with the other agent. Suppose Alice owns qutrits 1 and 2, Bob qutrit 3 and Charlie qutrit 4. The combined state of four particles is

$$|\Phi\rangle_{1234} = |P\rangle_1 \otimes |\psi\rangle_{234}, \tag{1}$$

where

$$|P\rangle_1 = \alpha|0\rangle_1 + \beta|1\rangle_1 + \gamma|2\rangle_1, \tag{2}$$

$$|\psi\rangle_{234} = \frac{1}{\sqrt{3}}(|000\rangle_{234} + |111\rangle_{234} + |222\rangle_{234}), \tag{3}$$

and α, β and γ are complex and satisfy $|\alpha|^2 + |\beta|^2 + |\gamma|^2 = 1$. Alice wants to send her arbitrary unknown single-qutrit state $|P\rangle_1$ in such a way that anyone of the two agents Bob and Charlie can reconstruct the unknown state with the other's help. In order to achieve her goal, Alice performs a generalized Bell-state projective measurement on her qutrit pair (1, 2). After Alice's measurement, the system's state evolves to one of the following nine possible results:

$$|\Psi_{00}\rangle_{12}\langle\Psi_{00}|\Phi\rangle = \frac{1}{3}|\Psi_{00}\rangle_{12}(\alpha|00\rangle_{34} + \beta|11\rangle_{34} + \gamma|22\rangle_{34}), \tag{4}$$

$$|\Psi_{01}\rangle_{12}\langle\Psi_{01}|\Phi\rangle = \frac{1}{3}|\Psi_{01}\rangle_{12}(\alpha|11\rangle_{34} + \beta|22\rangle_{34} + \gamma|00\rangle_{34}), \tag{5}$$

$$|\Psi_{02}\rangle_{12}\langle\Psi_{02}|\Phi\rangle = \frac{1}{3}|\Psi_{02}\rangle_{12}(\alpha|22\rangle_{34} + \beta|00\rangle_{34} + \gamma|11\rangle_{34}), \tag{6}$$

$$|\Psi_{10}\rangle_{12}\langle\Psi_{10}|\Phi\rangle = \frac{1}{3}|\Psi_{10}\rangle_{12}(\alpha|00\rangle_{34} + e^{-2\pi i/3}\beta|11\rangle_{34} + e^{-4\pi i/3}\gamma|22\rangle_{34}), \tag{7}$$

$$|\Psi_{20}\rangle_{12}\langle\Psi_{20}|\Phi\rangle = \frac{1}{3}|\Psi_{20}\rangle_{12}(\alpha|00\rangle_{34} + e^{-4\pi i/3}\beta|11\rangle_{34} + e^{-8\pi i/3}\gamma|22\rangle_{34}), \tag{8}$$

$$|\Psi_{11}\rangle_{12}\langle\Psi_{11}|\Phi\rangle = \frac{1}{3}|\Psi_{11}\rangle_{12}(\alpha|11\rangle_{34} + e^{-2\pi i/3}\beta|22\rangle_{34} + e^{-4\pi i/3}\gamma|00\rangle_{34}), \tag{9}$$

$$|\Psi_{21}\rangle_{12}\langle\Psi_{21}|\Phi\rangle = \frac{1}{3}|\Psi_{21}\rangle_{12}(\alpha|11\rangle_{34} + e^{-4\pi i/3}\beta|22\rangle_{34} + e^{-8\pi i/3}\gamma|00\rangle_{34}), \tag{10}$$

$$|\Psi_{12}\rangle_{12}\langle\Psi_{12}|\Phi\rangle = \frac{1}{3}|\Psi_{12}\rangle_{12}(\alpha|22\rangle_{34} + e^{-2\pi i/3}\beta|00\rangle_{34} + e^{-4\pi i/3}\gamma|11\rangle_{34}), \tag{11}$$

$$|\Psi_{22}\rangle_{12}\langle\Psi_{22}|\Phi\rangle = \frac{1}{3}|\Psi_{22}\rangle_{12}(\alpha|22\rangle_{34} + e^{-4\pi i/3}\beta|00\rangle_{34} + e^{-8\pi i/3}\gamma|11\rangle_{34}), \tag{12}$$

where

$$|\Psi_{nm}\rangle = \sum_{j=0}^{2} e^{2\pi ijn/3}|j\rangle \otimes |(j+m) mod3\rangle/\sqrt{3}, \quad n \in \{0,1,2\},\ m \in \{0,1,2\}. \tag{13}$$

For each possible result, the treatment is similar in this paper. As an enumeration, only one result is taken as an example hereafter. Without loss of generality, suppose Alice's measurement result is $|\Psi_{00}\rangle_{12}$. In this case, the qutrits 3 and 4 collapse to the entangled state

$$|K_1\rangle_{34} = \frac{1}{3}(\alpha|00\rangle_{34} + \beta|11\rangle_{34} + \gamma|22\rangle_{34}). \tag{14}$$

This state can be rewritten as

$$\begin{aligned} |K_1\rangle_{34} &= \frac{1}{3}(\alpha|00\rangle_{34} + \beta|11\rangle_{34} + \gamma|22\rangle_{34}) \\ &= \frac{1}{3}[\frac{1}{\sqrt{3}}|\xi_0\rangle_3(\alpha|0\rangle_4 + \beta|1\rangle_4 + \gamma|2\rangle_4) \\ &+ \frac{1}{\sqrt{3}}|\xi_1\rangle_3(\alpha|0\rangle_4 + e^{-2\pi i/3}\beta|1\rangle_4 + e^{-4\pi i/3}\gamma|2\rangle_4) \\ &+ \frac{1}{\sqrt{3}}|\xi_2\rangle_3(\alpha|0\rangle_4 + e^{-4\pi i/3}\beta|1\rangle_4 + e^{-2\pi i/3}\gamma|2\rangle_4)], \end{aligned} \tag{15}$$

where

$$\begin{aligned} |\xi_0\rangle &= \frac{1}{\sqrt{3}}(|0\rangle + |1\rangle + |2\rangle), \\ |\xi_1\rangle &= \frac{1}{\sqrt{3}}(|0\rangle + e^{2\pi i/3}|1\rangle + e^{4\pi i/3}|2\rangle), \\ |\xi_2\rangle &= \frac{1}{\sqrt{3}}(|0\rangle + e^{4\pi i/3}|1\rangle + e^{2\pi i/3}|2\rangle). \end{aligned} \tag{16}$$

The three states $\{|\xi_t\rangle, t = 0, 1, 2\}$ are related to the computation basis vectors $\{|0\rangle, |1\rangle, |2\rangle\}$, and form a complete orthogonal basis set of a single-qutrit Hilbert space. After her measurement, Alice publishes her result $|\Psi_{00}\rangle_{12}$ and assigns either Bob or Charlie (she makes the choice at random) to measure his qutrit in the complete orthogonal basis set proposed above. Without loss of generality, suppose Bob is assigned. If Bob's measurement result is $|\xi_0\rangle_3$, the qutrit 4 (in Charlie's possession) is projected onto $\alpha|0\rangle_4+\beta|1\rangle_4+\gamma|2\rangle_4$. This state is exactly the original state $|P\rangle$. Now Charlie cooperates with Bob to get his result over a public channel. With Bob's help, Charlie reconstructs the original state with no unitary operation. While Bob's measurement result is $|\xi_1\rangle_3$, the qutrit 4 is projected onto $\alpha|0\rangle_4 + e^{-2\pi i/3}\beta|1\rangle_4 + e^{-4\pi i/3}\gamma|2\rangle_4$. Charlie reconstructs the original state $|P\rangle$ by performing an unitary operation $U_1 = \sum_{j=0}^{2} e^{2\pi ij/3}|j\rangle\langle j|$ under Bob's help. Similarly, if Bob's measurement result is $|\xi_2\rangle_3$, the qutrit 4 is projected onto $\alpha|0\rangle_4 + e^{-4\pi i/3}\beta|1\rangle_4 + e^{-2\pi i/3}\gamma|2\rangle_4$. In this case, conditioned on Bob's classical bit for his result Charlie recovers the original state $|P\rangle$ by carrying out an unitary operation $U_2 = \sum_{j=0}^{2} e^{4\pi ij/3}|j\rangle\langle j|$. So far, we have demonstrated the three-party qutrit state sharing scheme of an arbitrary unknown single-qutrit state. Note that in the above scheme Alice assigns the agent Bob to measure the qutrit 3 and the agent Charlie to reconstruct the original state $|P\rangle$. While the agent who is assigned by Alice to do a single-qutrit measurement is Charlie and Bob is assigned to recover the quantum information. As the symmetry, the above procedure is also feasible. Here we do not state it anymore.

Now let's analyze the scheme security. We consider two kinds of eavesdroppers.

(a) Outside eavesdropper: Suppose there is an illegitimate user named Eve. She wants to gain the quantum information which Alice lets Bob and Charlie share. To achieve her goal, she entangles an ancilla with the quantum channel during the particle distribution process. For this case, the security check of the present three-party qutrit state sharing scheme is very similar to that of the protocol proposed by Hillery et al.[2]. That is, the security depends completely on whether the three legitimate users have securely shared the entangled GHZ state which are taken as the quantum channel. Here we briefly review the check method.

The legitimate user Alice choose randomly a single-qutrit measurement basis (MB) $\{|\xi_t\rangle, t = 0, 1, 2\}$ or $\{|0\rangle, |1\rangle, |2\rangle\}$ to measure her qutrit. After her measurement, Alice tells the other two legitimate users Bob and Charlie which MB she has chosen for her qutrit. Bob and Charlie use the same MB as Alice to measure their respective qutrit. Their measurement outcomes should be strongly correlated. If there exists an eavesdropper Eve in the quantum line, her operation will of course introduce some disturbance which will cause some qutrit errors. Thus, when the legitimate users publicly compare their results, they will find some incorrelation which means that the quantum channel is disturbed. Alternatively, there may exist an eavesdropper Eve. In this case, the quantum sharing process is aborted. Incidentally, in our scheme the qutrit GHZ state is assumed to be safely shared among legitimate users. This can be achieved using the quantum purification and distillation or quantum repeater techniques if the quantum channel noise or decoherence is taken into account[29-34].

(b) Inside eavesdropper: suppose one of two legitimate agents (say, Bob) is dishonest. He wants to solely and safely recover Alice's quantum information without any assistance from Charlie. To achieve his goal, Bob captures the qutrit Alice sends to Charlie and then sends Charlie a fake qutrit he has prepared before. In this case, only when Alice designates him to reconstruct the state, he can successfully get the state $|P\rangle$ and avoid the security detection. However, if Alice designates not Bob but Charlie to reconstruct the state, then the state reconstructed by Charlie will differ from the state Alice has sent. In this case, if Alice and Charlie publicly compare the state, the eavesdropping can be disclosed. Hence, the success probability for the dishonest Bob is only 50% in each run. During the whole sharing process, if the amount of the check state is large enough, then the dishonest Bob will be revealed.

III. Multi-party qutrit-state sharing scheme

Now let us generalize the three-party qutrit state sharing scheme to multi-party case. Suppose there are $N+1$ legitimate users. Alice is the quantum information sender. By the way, the quantum information is still given by the equation 1. The other N users are Alice's agents, named as Bob (1st agent), Charlie (2nd agent),..., Zach (Nth agent), respectively. All the legitimate users have successfully shared in advance a general $(N+1)-$qutrit GHZ state

$$|\psi'\rangle_{23...(N+2)} = \frac{1}{\sqrt{3}}(|00...0\rangle_{23...(N+2)} + |11...1\rangle_{23...(N+2)} + |22...2\rangle_{23...(N+2)}). \tag{17}$$

Qutrit 2 belongs to Alice, and qutrits 3, 4,...$(N+2)$ to Bob, Charlie,...Zach, respectively. Similarly, in order to split her quantum information into N parts for her N agents, Alice performs a generalized Bell-state projective measurement on her qutrit pair (1,2) and publishes her result. After the generalized Bell-state projective measurement, the system's state evolves to one of the following nine possible results:

$$|\Psi_{00}\rangle_{12}\langle\Psi_{00}|\Phi'\rangle = \frac{1}{3}|\Psi_{00}\rangle_{12}(\alpha|00...0\rangle_{34...(N+2)} + \beta|11...1\rangle_{34...(N+2)} + \gamma|22...2\rangle_{34...(N+2)}), \tag{18}$$

$$|\Psi_{01}\rangle_{12}\langle\Psi_{01}|\Phi'\rangle = \frac{1}{3}|\Psi_{01}\rangle_{12}(\alpha|11...1\rangle_{34...(N+2)} + \beta|22...2\rangle_{34...(N+2)} + \gamma|00...0\rangle_{34...(N+2)}), \tag{19}$$

$$|\Psi_{02}\rangle_{12}\langle\Psi_{02}|\Phi'\rangle = \frac{1}{3}|\Psi_{02}\rangle_{12}(\alpha|22...2\rangle_{34...(N+2)} + \beta|00...0\rangle_{34...(N+2)} + \gamma|11...1\rangle_{34...(N+2)}), \tag{20}$$

$$|\Psi_{10}\rangle_{12}\langle\Psi_{10}|\Phi'\rangle = \frac{1}{3}|\Psi_{10}\rangle_{12}(\alpha|00...0\rangle_{34...(N+2)} + e^{-2\pi i/3}\beta|11...1\rangle_{34...(N+2)} + e^{-4\pi i/3}\gamma|22...2\rangle_{34...(N+2)}), \tag{21}$$

$$|\Psi_{20}\rangle_{12}\langle\Psi_{20}|\Phi'\rangle = \frac{1}{3}|\Psi_{20}\rangle_{12}(\alpha|00...0\rangle_{34...(N+2)} + e^{-4\pi i/3}\beta|11...1\rangle_{34...(N+2)} + e^{-8\pi i/3}\gamma|22...2\rangle_{34...(N+2)}), \tag{22}$$

$$|\Psi_{11}\rangle_{12}\langle\Psi_{11}|\Phi'\rangle = \frac{1}{3}|\Psi_{11}\rangle_{12}(\alpha|11...1\rangle_{34...(N+2)} + e^{-2\pi i/3}\beta|22...2\rangle_{34...(N+2)} + e^{-4\pi i/3}\gamma|00...0\rangle_{34...(N+2)}), \quad (23)$$

$$|\Psi_{21}\rangle_{12}\langle\Psi_{21}|\Phi'\rangle = \frac{1}{3}|\Psi_{21}\rangle_{12}(\alpha|11...1\rangle_{34...(N+2)} + e^{-4\pi i/3}\beta|22...2\rangle_{34...(N+2)} + e^{-8\pi i/3}\gamma|00...0\rangle_{34...(N+2)}), \quad (24)$$

$$|\Psi_{12}\rangle_{12}\langle\Psi_{12}|\Phi'\rangle = \frac{1}{3}|\Psi_{12}\rangle_{12}(\alpha|22...2\rangle_{34...(N+2)} + e^{-2\pi i/3}\beta|00...0\rangle_{34...(N+2)} + e^{-4\pi i/3}\gamma|11...1\rangle_{34...(N+2)}), \quad (25)$$

$$|\Psi_{22}\rangle_{12}\langle\Psi_{22}|\Phi'\rangle = \frac{1}{3}|\Psi_{22}\rangle_{12}(\alpha|22...2\rangle_{34...(N+2)} + e^{-4\pi i/3}\beta|00...0\rangle_{34...(N+2)} + e^{-8\pi i/3}\gamma|11...1\rangle_{34...(N+2)}). \quad (26)$$

This means that Alice can get anyone of the nine possible results. Similar to the three-party case, without loss of generality we only take one result as an enumeration hereafter. Suppose Alice's measurement result is $\Psi_{00}\rangle_{12}$. In this case, the state of the qutrits $3, 4, ..., (N+2)$ is

$$|K_2\rangle_{34...(N+2)} = \frac{1}{3}(\alpha|00...0\rangle_{34...(N+2)} + \beta|11...1\rangle_{34...(N+2)} + \gamma|22...2\rangle_{34...(N+2)}). \quad (27)$$

This state can be reexpressed as

$$\begin{aligned}
& |K_2\rangle_{34...(N+2)} \\
= & \frac{1}{3}(\alpha|00...0\rangle_{34...(N+2)} + \beta|11...1\rangle_{34...(N+2)} + \gamma|22...2\rangle_{34...(N+2)}) \\
= & (\frac{1}{\sqrt{3}})^{N+1}\sum_{l_1=0}^{2}\sum_{l_2=0}^{2}...\sum_{l_{m-1}=0}^{2}\sum_{l_{m+1}=0}^{2}...\sum_{l_N=0}^{2} \qquad (28) \\
\times & [|\xi_{l_1}\rangle_3|\xi_{l_2}\rangle_4...|\xi_{l_{m-1}}\rangle_{m+1}(\alpha|0\rangle_{m+2} + \beta e^{-2\pi iL/3}|1\rangle_{m+2} + \gamma e^{-4\pi iL/3}|2\rangle_{m+2})|\xi_{l_{m+1}}\rangle_{m+3}...|\xi_{l_N}\rangle_{N+2}],
\end{aligned}$$

where

$$L = \sum_{i=1}^{m-1} l_i + \sum_{j=m+1}^{N} l_j. \quad (29)$$

Alice can assign any agent to reconstruct the unknown state. In other words, anyone of the N agents has the chance to reconstruct the unknown state. After Alice's assignment, all the other agents should perform some operations and then help the assigned agent to reconstruct the state. Without loss of generality, we assume Alice assigns the mth agent to reconstruct her original state. According to the equation 28, after the other $N-1$ agents' measurement the qutrit in the mth agent's possession is left into $\alpha|0\rangle_{m+2} + \beta e^{-2\pi iL/3}|1\rangle_{m+2} + \gamma e^{-4\pi iL/3}|2\rangle_{m+2}$. If all the other agents collaborate with the assigned agent, he/she can reconstruct the original state $|P\rangle$ in his/her qutrit by performing the unitary transformation $U_3 = \sum_{j=0}^{2} e^{2\pi ijL/3}|j\rangle\langle j|$.

The security of the multi-party qutrit state sharing scheme is same as the security of the three-party qutrit state sharing scheme: any eavesdropping leads to the discrepancy between the state that Alice sends and the state that legitimate user reconstructs. Thus an eavesdropping can be detected by publicly comparing a subset of the quantum states.

IV. Conclusion

In summary, an interesting work for quantum state sharing of an arbitrary unknown single-qutrit state has been done. In this paper, we take a general GHZ state as the quantum channel. The state sender Alice performs a generalized Bell-state projective measurement and publishes her measurement result. As the symmetry, anyone (the receiver) of the N agents can regenerate the original state when he/she collaborates with the others. The other agents are required to perform one single-qutrit measurement on their respective qutrit. Conditioned on Alice's two classical bits, the receiver can reconstruct the original state via an unitary transformation only when he/she gets the other agents' help. In addition, we use multi-qutrit entangled stat, instead of EPR pairs, as the entangled quantum system, and it is more useful in maintaining security.

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